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Fractal patterns of cracks during a dynamic process

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Abstract. In this paper we have analysed a simple crack propagation lattice model which incorporates the bond-breaking probability $P_i \propto (\delta_i - \delta_c)^\eta$, where δ_i is the ith bond elongation and δ_c is the bond-breaking threshold below which the bond is unbroken. A two-dimensional triangular lattice has been employed with nearest-neighbour central forces. The cases of a uniform small dilation strain and of a small shear in the horizontal direction with different exponents η have been considered. We have found that the crack patterns during the breaking process are fractal structures with the fractal dimensions depending on the bond-breaking probability exponent η in a limited large lattice.

Fracture phenomena are some of the most interesting processes in materials science. It is quite difficult to understand fracture processes on a fundamental level. Since it was first pointed out by Mandelbrot *et al* (1984) that cracks are fractals in nature, many new approaches including computer simulations and experiments have been devoted to the fractal features of the fracture of material quite extensively (Herrmann and Roux 1990); in general, computer simulations of fracture phenomena are divided into two, i.e. the deterministic growth of a fractal crack by percolation-type models and the probabilistic growth of a fractal crack using the DLA (Witten and Sander 1981) as the guideline, which ranges from a random resistor network (Duxbury *et al* 1986, Beale and Srolovitz 1988, Herrmann 1989, Herrmann *et al* 1989) to fracture surfaces (Peng and Tian 1990, Cao *et al* 1991), to crack-growth models (Louis and Guinea 1987, Hinrichsen *et al* 1989, Meakin *et al* 1989).

In the following we shall consider a simple lattice model which assumes the elastic medium to be represented by a network of nodes joined by Hookean springs (Duxbury *et al* 1986, Beale and Srolovitz 1988, Herrmann 1989, Herrmann *et al* 1989). Suppose that the medium network is perfect, which means that no crack exists before the beginning of the simulation, and we wish to study how cracks are formed.

The candidates for breaking bonds are first selected to be those bonds whose elongations δ_i are larger than the breaking threshold δ_c; then a bond-breaking probability is assigned to each candidate:

$$P_i \propto (\delta_i - \delta_c)^\eta \tag{1}$$

where η is an adjustable probability exponent. The bond-breaking condition probability proposed is different from those of Herrmann (1989) and Meakin *et al* (1989). Here we have introduced δ_c and the breaking probability is a function of the extra elongation of a bond compared with the threshold. If all unbroken bonds have

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an elongation $\delta_i \gg \delta_c$, then $P_i \propto \delta_i^{\eta}$ (Meakin *et al* 1989); if their elongation $\delta_i \leqslant \delta_c$, then the breaking process will be stopped. Moreover, the breaking candidates for bonds are not restricted to the surfaces of the 'crack' formed; instead, all bonds (except for broken bonds) are being examined in every step of the simulation.

We adopt a two-dimensional triangular network of nodes as depicted in figure 1, connected by Hookean springs. The lattice consists of 150 rows and 150 nodes; 149 nodes interchange in each row. At the start of each simulation, each of the nodes (except for those at the edge of the lattice) is connected to six nearest neighbours. Two cases are considered: isotropic dilation, i.e. the components of displacement of any node are $U_{ix} = \alpha X_i$, $U_{iy} = \alpha Y_i$, where X_i, Y_i are the coordinates of the ith node; shear along the horizontal direction by the coordinate transformation of nodes $(X_i, Y_i) \rightarrow (X_i + \alpha Y_i, Y_i)$, i.e. $U_{ix} = \alpha Y_i$, $U_{iy} = 0$, where $\alpha = 10^{-2}$. In all simulations the bond force constant $k_{ij} = k = 20$; $k_{ij} = 0$ if the bond is a broken bond.

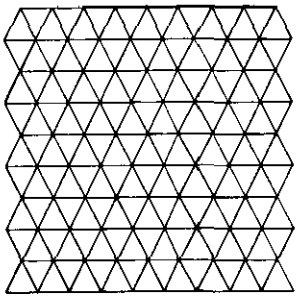

Figure 1. A typical two-dimensional triangular lattice.

For the network system the elastic energy E can be written as follows:

$$E = \tfrac{1}{2} \sum_{ij} k_{ij} [(\boldsymbol{U}_i - \boldsymbol{U}_j) \cdot \boldsymbol{r}_{ij}]^2 \tag{2}$$

where the sum index j runs over all the nearest nodes of the ith node, $\boldsymbol{U}_i$ and $\boldsymbol{U}_j$ are the displacements of the ith and jth nodes, and $\boldsymbol{r}_{ij}$ is the unit vector in the direction of the bond joining these nodes (Meakin *et al* 1989).

Table 1. Results of two simulations.

Exponent	Stress field type	D
0.5	Dilation	1.48 ± 0.03
1.0	Dilation	1.49 ± 0.02
1.5	Dilation	1.54 ± 0.03
2.0	Dilation	1.73 ± 0.02
0.5	Shear	1.26 ± 0.03
1.0	Shear	1.28 ± 0.03
1.5	Shear	1.31 ± 0.05
2.0	Shear	1.41 ± 0.05

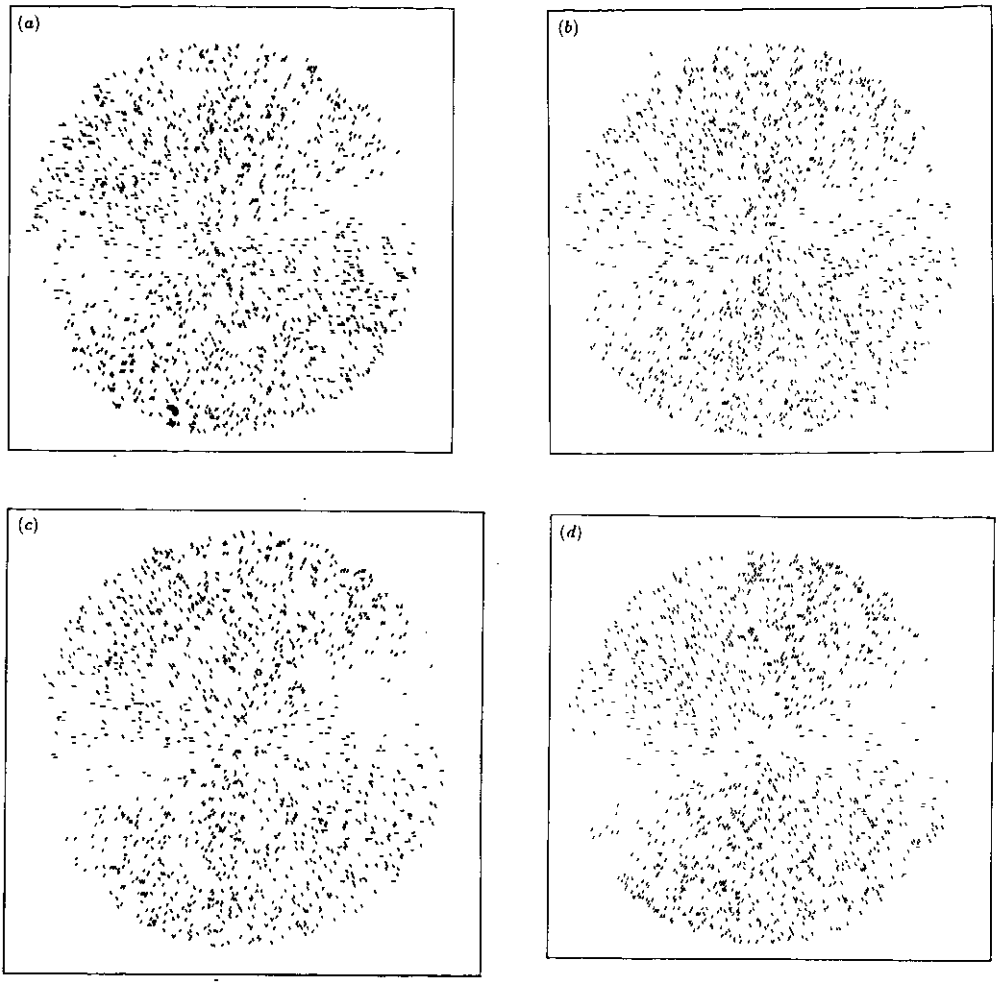

Figure 2. Crack distribution with a small dilation strain for various values of the probability exponent η (the number of broken bonds is 1300 in every case): (*a*) $\eta = 0.5$; (*b*) $\eta = 1.0$; (*c*) $\eta = 1.5$; (*d*) $\eta = 2.0$.

At the beginning of each simulation, the bond threshold strain is typically given by $\delta_c = 0.5 \times 10^{-3}$ unit length (we assume the lattice constant to be one unit). Such a threshold and magnitude of applied displacement ensure that the crack can always be formed at the beginning of the simulation, since we have chosen $\alpha > \delta_c$. After isotropic dilation or shear of the lattice, select all those bonds whose elongations δ_i are larger than δ_c; then break one of them according to the probability given by $P_i / \sum_i P_i$ (the index i runs over all unbroken bonds). If a bond of the network is broken, this means that a crack is formed and the system is relaxed by solving the equations of elastic energy minimum to mechanical equilibrium using standard block overrelaxation methods (Jenning 1977). The relaxation procedure was an iteration process in the computational realization. When for the whole system $|U_{it}^{(k)} - U_{it}^{(k-1)}| \leqslant 10^{-4}$ (Meakin *et al* 1989, Peng and Tian 1990, Cao *et al* 1991) for any node i and for $t = x$ or y, the relaxation process was stopped, and we

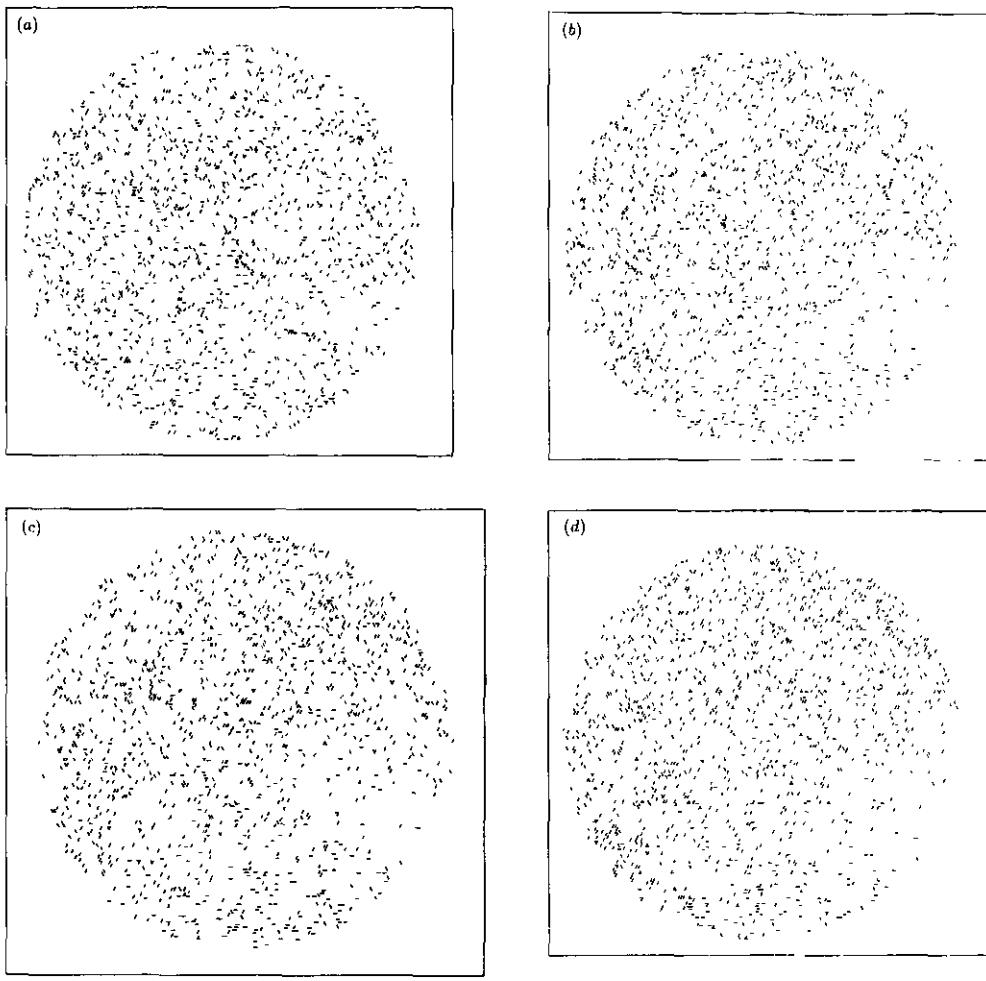

Figure 3. Crack distribution with a small shear strain in the horizontal direction for various values of the probability exponent η (the number of broken bonds is 1300 in every case): (*a*) $\eta = 0.5$; (*b*) $\eta = 1.0$; (*c*) $\eta = 1.5$; (*d*) $\eta = 2.0$.

repeated the above sequence of random bond breaking and relaxation events (only for $\delta_i > \delta_c$). Throughout the simulation, the positions of the nodes at the edges of the network are fixed. In all simulations, if the number of broken bonds reaches 1300, the simulation is stopped. In order to overcome the effect of the edge of the system and to use the block overrelaxation method, we restrict our simulations to be localized in the centre region, which means that no broken bonds are allowed to occur at the edge of the network, i.e. a breaking threshold larger than δ_c was selected to exist for the boundary region bonds; so the distribution of cracks has a somewhat circular geometry. Here the free-boundary conditions were adopted in our simulation (Louis and Guinea 1987, Meakin *et al* 1989, Peng and Tian 1990). Although in the horizontal direction a periodic boundary condition could have some influence on the simulation results quantitatively, it would not change the results qualitatively (Hinrichsen *et al* 1989, Cao *et al* 1991). The results obtained from typical simulations

carried out with isotropic dilation and four values of 0.5, 1.0, 1.5 and 2.0 for the probability exponent η are shown in figures 2(*a*), 2(*b*), 2(*c*) and 2(*d*), respectively. It is evident from these figures that four η-values lead to similar random structures, but the different η-values have influenced the crack distribution in a quantitative way. Figures 3(*a*), 3(*b*), 3(*c*) and 3(*d*) show the results obtained from four values of 0.5, 1.0, 1.5 and 2.0 of the exponent for the case of shear strain.

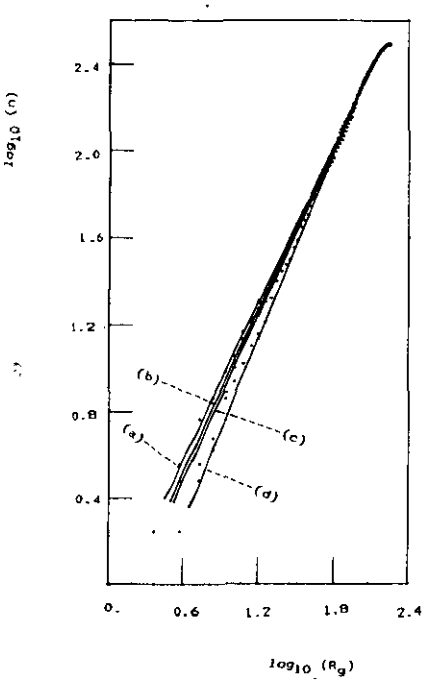

Figure 4. Dependence of the gyration radius R_g on the number n of broken bonds for uniform dilation: curve (*a*), $\eta = 0.5$; curve (*b*), $\eta = 1.0$; curve (*c*), $\eta = 1.5$; curve (*d*), $\eta = 2.0$.

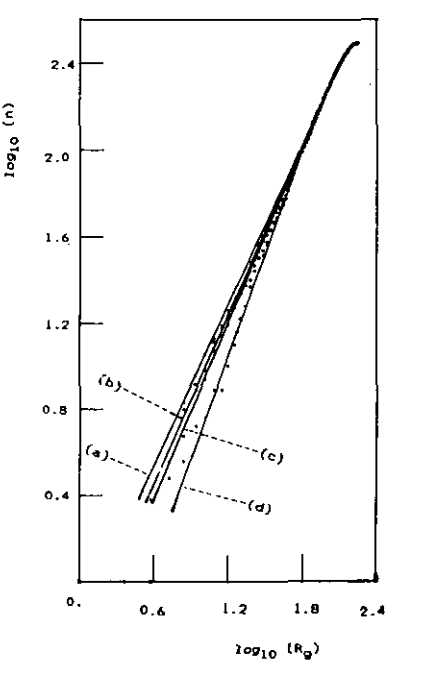

Figure 5. Dependence of the gyration radius R_g on the number n of broken bonds for a small shear in the horizontal direction: curve (*a*), $\eta = 0.5$; curve (*b*), $\eta = 1.0$; curve (*c*), $\eta = 1.5$; curve (*d*), $\eta = 2.0$.

All our estimates for the fractal dimensionality are based on the observation of a power-law relationship between the radius R_g of gyration of the crack (broken bonds) measured on the unstrained network (R_g) and the number n of broken bonds (Hinrichsen *et al* 1989, Meakin *et al* 1989):

$$R_g \propto n^{\beta} \tag{3a}$$

$$D = 1/\beta. \tag{3b}$$

We have obtained the fractal dimensionalities from equation (3) according to the results of two simulations for each exponent as shown in table 1.

From table 1 and figures 2 and 3, we have found that the exponents η have an effect on the crack distribution, that the fractal dimensions of cracks can be increased monotonically for two cases by increasing η and that the fractal dimensionalities of

cracks generated using dilation are larger than those obtained using the shear method, which differs from the results of Meakin *et al* (1989) where D increases continuously as η is decreased. In the case of dilation, the shapes of cracks are found to be more closed along the vertical direction with the parameter increasing; this crack tendency is clearer than that in the shear case. In all simulations the dependence of $\log_{10} R_g$ versus $\log_{10} n$ is a linear relation, as shown in figure 4 and figure 5. From the log–log plots, in the asymptotic region, i.e. $R_g \to \infty$, the exponent D becomes independent of exponent η: $D \simeq 1.5$ for the dilation case and $D \simeq 1.3$ for the shear case. We do not know the real reason for this; it may be due to the change in the value of η which has only a local effect in a larger network, and a larger exponent η might produce more reasonable results. We are at present researching this aspect.

In conclusion we have presented a simple dynamic model to study crack formation and distribution. We also characterized quantitatively crack patterns by means of the concept of fractal geometry. We found that the fractal dimensions D depend on the power η in a limited large network and, for a gyration radius $R_g \to \infty$, D had specific values for the dilation and shear cases, respectively; the crack tendency is clearer for uniform dilation than for shear. We admit that our model may be only a crude representation of real experimental systems. Nevertheless, our results may be useful in further studies of fracture and cracks.

Acknowledgments

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